## 2002 Sun Microsystems and TopCoder Collegiate Challenge - Problem Statement

## SetPartition PROBLEM STATEMENT

Set partitions divide elements $A, B, .,<N-t h$ letter $>$ into non-empty subsets. For example, when $N=4$ (the set is $A B C D$ ), there are fifteen distinct partitions: $\{A B C D\},\{A B C, D\},\{A B D, C\},\{A B, C D\},\{A B, C, D\},\{A C D, B\},\{A C, B D\},\{A C, B, D\},\{A D, B C\}$, $\{A, B C D\},\{A, B C, D\},\{A D, B, C\},\{A, B D, C\},\{A, B, C D\}$, and $\{A, B, C, D\}$. One way to define a partition is through a partitioning string. Each element of a partitioning string specifies the number of the subset to which the corresponding element of the set goes. For example, partitioning string $\{0,1,1,2,1\}$ specifies the $\{A, B C E, D\}$ partition of $A B C D E$ the first position specifies the subset number for $A$, the second position specifies the subset number for $B$, and so on. Therefore, A goes to subset 0, B,C, and E go to subset 1, and D goes to subset 2.

For a string to be a valid partitioning string, all its elements must be nonnegative, its initial element must be 0, and the following limiting relation must hold for all i $>0$ : Ai $<=1+\max (A 0 . . A i-1)$. For example, $\{1,0\}$ is not a valid partitioning string because it does not start with $0,\{0,-1\}$ is invalid because it has negative numbers, and $\{0,3,1,2\}$ is invalid because its second element violates the limiting relation. Note that there is a one-to-one correspondence between the partitioning strings and the set partitions.
You can order the partitions by ordering their corresponding partitioning strings. A natural order for strings is lexicographic, like words in a dictionary. For example, in lexicographic order $\{0,0,1,2\}$ comes before $\{0,1,0,0\}$, but after $\{0,0,1,1\}$. If you order all possible partitioning strings of length $N$, $\{0,0,0, \ldots, 0\}$ would be the first, and $\{0,1,2, \ldots, N-1\}$ would be the last partitioning string. The partitions of $A B C D$ in the example at the top of the problem are given in lexicographic order of their corresponding partitioning strings.
Write a method that, given a set partition, finds the set partition corresponding to the next partitioning string.

DEFINITION

Class Name: SetPartition
Method Name: nextPartition
Parameters: String[]
Returns: String[]
Method signature (be sure your method is public): String[] nextPartition(String[] partition);

TopCoder will ensure that:

- partition has between 1 and 25 elements, inclusive,
- Each element of the partition has between 1 and 26 elements, inclusive,
- Each element of the partition consists only of characters 'A' through 'Z', inclusive,
- Elements of partition and characters inside each element are sorted alphabetically in ascending order (this ensures that the partitioning string of the input partition is valid),
- There are no duplicate characters in the partition (this rule works across all elements),
- If a character <ch> is listed in an element of the partition, all characters from 'A' to <ch>, inclusive, are also listed, possibly in another element (this ensures that there are no gaps in the initial set),
- At least one element of the partition has two or more characters (this ensures that the next lexicographic partitioning string exists).

EXAMPLES

1. partition=\{"AB", "C", "D"\}. The corresponding partitioning string is $\{0,0,1,2\}$; the next partitioning string in lexicographic order is $\{0,1,0,0\} ;$ your method should return $\{$ "ACD", "B"\}.
2. partition=\{"ABC", "DEF"\}. The corresponding partitioning string is
$\{0,0,0,1,1,1\} ;$ the next partitioning string is $\{0,0,0,1,1,2\}$; your method should return \{"ABC", "DE", "F"\}.
3. If partition=\{"ADFHKM", "BZ", "CXY", "EOPVW", "GLN", "IJSTU", "Q", "R"\}, your method should return \{"ADFHKM", "B", "CXYZ", "EOPVW", "GLN", "IJSTU", "Q", "R"\}. 4. If partition=\{"A","B","C","D","E","FG"\}, your method should return \{"A", "B", "C", "D", "E", "F", "G"\}.
4. If partition=\{"ABCDEFG"\}, your method should return \{"ABCDEF","G"\}.
